

The Twelfth East Asia-Pacific Conference on Structural Engineering and Construction

## Modeling and Analyzing of Hysteresis Behavior of Magneto Rheological Dampers

X.C. Guan<sup>a\*</sup>, P.F. Guo<sup>a</sup>, J.P. Ou<sup>b</sup>

<sup>a</sup>*School of Civil Engineering, Harbin Institute of Technology, China*

<sup>b</sup>*School of Civil Engineering, Dalian University of Technology, China*

---

### Abstract

Damping force-velocity hysteresis of a magnetorheological (MR) damper under sinusoidal displacement excitation is not only a typical indication of its dynamic performance, but also the foundation upon which a practical control strategy is established. Although numerous parametric and non-parametric models are effectively in predicting the hysteresis, their accuracy strongly depends on specific experimental data. Furthermore, little design guiding information can be explored from these models. With compressibility of MR fluid considered, ordinary differential equations (ODEs) of a physical MR damper model are derived in this paper. Then the corresponding lumped parameter model is developed, that is, a quasi-static MR model connected in series with a spring expressing compression of MR fluid. Moreover, the spring stiff expression is found to be equivalent to the “oil spring” in hydraulic technology. Decomposing a quasi-static MR model further to a friction element and a parallel-connected viscous element, these two basic elements in combination with a spring together fundamentally constitute a dynamic MR damper model. Consequently, a clear developing process of the hysteresis can be described with these three basic elements. As another main contribution of this paper, expression for calculating hysteresis width is derived by neglecting viscous element, and dynamic design method of MR dampers is proposed.

© 2011 Published by Elsevier Ltd. Open access under [CC BY-NC-ND license](#).

### Selection

**Keywords:** MR damper, hysteresis, oil spring, physical model, dynamic design

---

---

\* Corresponding author  
Email: [guanxch@hit.edu.cn](mailto:guanxch@hit.edu.cn)

## 1. Introduction

In addition to representing dynamic performance, hysteresis of relationship between damping force and velocity of a magnetorheological (MR) damper under sinusoidal displacement excitation also plays an important role in building an effective control algorithm. To predict this hysteresis, different parametric and non-parametric models are developed, which mainly include: nonlinear Bingham plastic model (Weng et al., 2000; Choi et al., 2002), modified Bingham plastic model (Zhou and Qu, 2002), hysteretic bi-viscous model (Wereley et al., 1998; Wang and Meng, 2001), modified bi-viscous model (Li et al., 2009), nonlinear viscous elastic plastic model (Kamath and Wereley, 1997), Bouc-Wen model (Dyke et al., 1996), modified Bouc-Wen model (Yang, 2001; Jansen et al., 2000; Wang et al., 2006; Gao et al., 2004), temperature phenomenal model with mass element (Xu et al., 2005), polynomial model (Choi et al., 2005), Dahl model (Zhou et al., 2005), sigmoid function model (Ma et al., 2002; Wang et al., 2003), modified sigmoid function model (Jiang and Li, 2008), neural networks model (Wang and Liao, 2004; Du et al., 2006; Wang et al., 2007; Wang et al., 2009), neuro-fuzzy model (Schurter and Roschke, 2000; Wilson and Abdullah, 2005; Gao and Wang, 2008). However, these models can't physically describe the hysteresis because their parameters are identified by fitting experimental data.

Peel et al. (1996) and Sims et al. (1999, 2000) extended their quasi-static electrorheological (ER) models by including dynamic effects accounting for the hysteric behavior of ER long stroke dampers. Based on material and geometry properties, a lumped parameter model consisting of a spring, a mass, a damper connected serially is developed in their studies. However, some model parameters such as stiffness, yield stress still need to be identified using experimental data.

Wang and Gordaninejad (2007) developed the first MR physical model by taking account of the compression of MR fluid and successfully simulated the hysteresis. In a similar way, under the assumption of constant fluid bulk modulus, ordinary differential equations (ODEs) of a physical MR model will be derived, and corresponding straightforward mechanical analogy will be also made. A practical dynamic design method will be proposed in the end of this paper for MR dampers under sinusoidal displacement excitation.

## 2. Modeling of The Hysteresis of MR Dampers

According to the definition of bulk modulus, compression of a fluid can be computed by:

$$\Delta V = \frac{\Delta P}{e} V \quad (1)$$

Where,  $\Delta P$  is change of pressure.  $V$  and  $e$  are respectively volume and modulus of fluid.

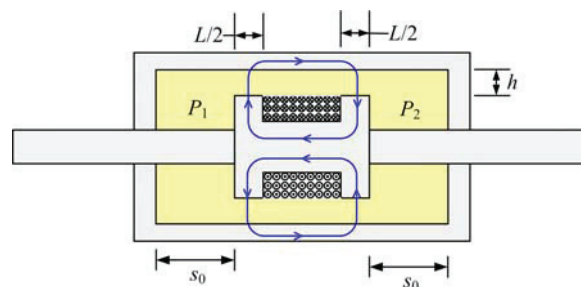


Figure 1: Typical structure of a MR damper

Then, flow rate loss due to the compression of fluid is:

$$q = \lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t} = \frac{dP}{dt} \frac{V}{e} \quad (2)$$

As shown in Figure 1, assuming that left and right chambers have the same constant fluid bulk modulus, then respective flow rate are:

$$Q_1 = uA - \frac{dP_1}{dt} \frac{V_1}{e} \quad (3.a)$$

$$Q_2 = uA + \frac{dP_2}{dt} \frac{V_2}{e} \quad (3.b)$$

Neglecting compression of fluid in the gap yields:

$$Q_1 = Q_2 = \text{sign}(P_1 - P_2)Q = \bar{Q}b \quad (4)$$

Where,  $b$  is average circumference of gap or the width of equivalent parallel plates.  $\bar{Q}$  is magnitude of flow rate passing through a unit wide parallel plates, and given by a quasi-static model as bellow (Guan and Guo, 2009).

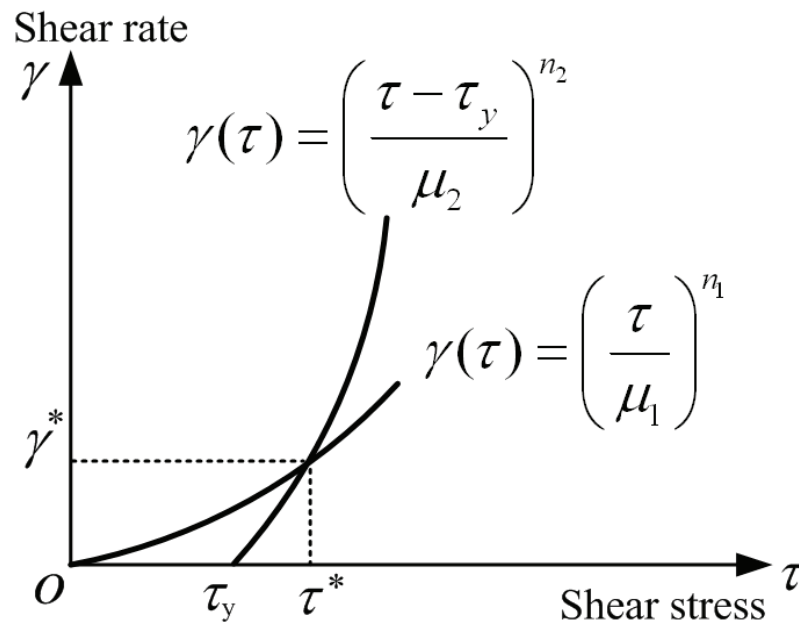


Figure 2: A general material model of MR fluid

$$\bar{Q} = \begin{cases} \frac{2^{-1-n_1} h^2 \left( \frac{P_{12} h}{\mu_1 L} \right)^{n_1}}{2+n_1} & 0 \leq P_{12} \leq P^* \\ \frac{2L^2}{P_{12}^2} \left( \frac{\mu_1^{-n_1} (\tau^*)^{2+n_1}}{2+n_1} + \frac{\mu_2 (\gamma^*)^{1+n_2} (\mu_2 \gamma^* - (2+n_2) \tau^*)}{2+3n_2+n_2^2} \right) & P_{12} > P^* \\ + \frac{(hP_{12} + 2L\mu_2 \gamma^* - 2L\tau^*) (h(1+n_2)P_{12} + 2L(-\mu_2 \gamma^* + \tau^*)) \left( \gamma^* + \frac{hP_{12} - 2L\tau^*}{2L\mu_2} \right)^{n_2}}{4L^2(1+n_2)(2+n_2)} & \end{cases} \quad (5)$$

Where  $P_{12}$  is pressure drop.  $P^* = 2\tau^*/L$ .  $L$  and  $h$  are effective length of piston and gap width respectively. As shown in Figure 2,  $\mu_1$  and  $\mu_2$  are pre-yield and post-yield viscosity.  $n_1$  and  $n_2$  are pre-yield and post-yield flow index.  $\tau_y$  and  $\tau^*$  are static and dynamic shear yield strength.  $\gamma^*$  dynamic shear yield rate. Friction force  $F_f$  is equivalent to the shear yield stress for a damper in the absence of magnetic field as:

$$\tau^*(0) = \frac{F_f}{A} \frac{h}{2L} \quad (6)$$

From Equations (3.a) and (3.b), change rate of pressure can be gained as:

$$\frac{dP_{12}}{dt} = \frac{d(P_1 - P_2)}{dt} = \frac{dP_1 - dP_2}{dt} = (uA - Q) \frac{e}{V_1 + V_2} \quad (7)$$

Noticing that  $V_1 = s_0 - sA$  and  $V_2 = s_0 + sA$ , above equation can be rewritten as:

$$u - \frac{dP_{12}}{dt} \Big/ k = \frac{Q}{A} \quad (8)$$

$$\text{where, } k = \frac{2s_0}{s_0^2 - s^2} e.$$

The first term on the left side of Equation (8) is excitation velocity; the second term is velocity loss due to the compression of fluid, viz., velocity generating only fluid compression of fluid but no flow rate; consequently the right side term act as an effective piston velocity  $u_e$ , viz., piston velocity in the case of incompressible fluid.

From Figure 3, it is obvious that effective velocity ( $u_e$ ) or damping force ( $F_d$ ) lags behind the excitation velocity ( $u$ ) due to the existence of a spring.

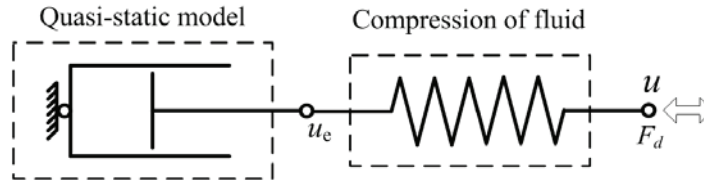


Figure 3: Mechanical element analogy of a MR damper model

Actually, the following derivation shows that spring stiff  $k$  in Equation (8) is equivalent to the stiff of oil spring in hydraulic technology,  $k_{oil}$ :

$$k_{oil} = \frac{F_d}{s} = e \left( \frac{A_1^2}{V_1} + \frac{A_2^2}{V_2} \right) = e \left( \frac{A_1^2}{(s_0 - s)A_1} + \frac{A_2^2}{(s_0 + s)A_2} \right) \quad (9)$$

Let  $A_1 = A_2$ , then above equation is rewritten as:

$$k_{oil} = kA = \frac{2s_0 A}{s_0^2 - s^2} e \quad (10)$$

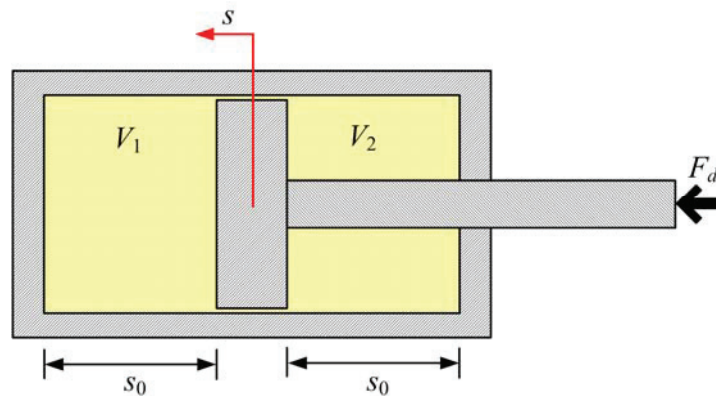


Figure 4: Oil spring of a hydraulic cylinder

### 3. Analyzing of the Hysteresis

#### 3.1. Fundamental mechanical elements for modeling the hysteresis

Since damping force of a MR damper is composed of a coulomb force  $F_\tau$  and a viscous force  $F_\eta$ , mechanical model in Figure 3 can be further decomposed to three basic elements as shown in Figure 5, in which parallel connected viscous element and friction element constitute a quasi-static model. Then, development of damping force-velocity hysteresis can be described with these three basic mechanical elements.

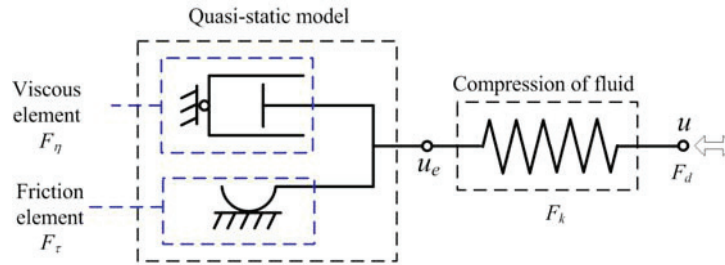


Figure 5: Basic elements constituting the mechanical analogy of a MR damper

### 3.2. Calculation of the hysteresis

Study in the previous section suggests that a spring and a friction element are two fundamental mechanical elements to model the hysteresis of MR dampers, and then mechanical model in Figure 5 can be simplified to the model in Figure 6 to calculate the hysteresis.

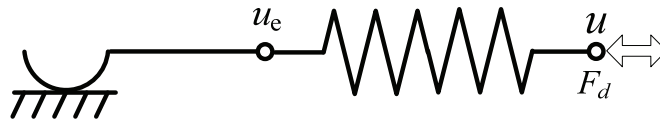


Figure 6: Simplified model of a MR damper with viscous element neglected

For the model in Figure 6, lag time of damping force behind excitation velocity,  $\delta$  can be computed by:

$$\int_{T/4}^{T/4+\delta} k(t)u(t)dt = -2P^* \quad (11)$$

And corresponding hysteresis width is:

$$du = u\left(\frac{T}{4}\right) - u\left(\frac{T}{4} + \delta\right) \quad (12)$$

It should be noted that they are independent of piston area and only functions of spring stiff for a given sinusoidal displacement.

## 4. Dynamic Design of MR Dampers

Up to now, MR dampers are generally static force oriented design based on quasi-static models. With preceding Equations (11) and (12), dynamic design can be conveniently performed for MR dampers under sinusoidal displacement excitations.

Since the lag time and hysteresis width are independent of effective area of piston, dynamic design and regular static design can be conducted in a relatively separate way, as shown in Figure 7. The procedure goes in detail as bellows:

- (1) Regular static design: Determine the sliding force  $F_\tau$  according to Equation (5).
- (2) Dynamic design: Replacing  $L/h$  in Equations (11) and (12) with  $F_\tau/A$ , solve hysteresis width for different strokes and effective areas of piston,  $du(s_0, A)$ , and choose a desirable pair of  $(s_{0is}, A_i)$ .
- (3) Choose appropriate  $L$  and  $h$  based mainly on magnetic analysis.

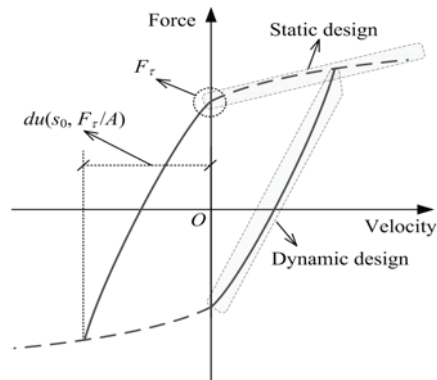


Figure 7: Dynamic design of MR dampers

## 5. Conclusions

To predict damping force-velocity hysteresis of MR dampers under sinusoidal displacement excitation, a physical model is developed by considering compressibility of MR fluid. Straightforward mechanical analogy is presented and a further simplified analogy model in which viscous element is neglected permits derivation of a formula for estimating the hysteresis width. With the help of this formula, dynamic design method of MR dampers is proposed at the end of this paper. It is found that the hysteresis width of a MR damper is independent of piston area and only a function of spring stiffness for a given sinusoidal displacement.

## Acknowledgments

This research is financially supported by the National Natural Science Foundation of China under grant 90815027, High-Tech Research and Development of China under grant 2006AA03Z103, National Basic Research Program of China under grant 2007CB714204, Commonwealth Scientific Research Program of China under grant 2008419073, and National Science and Technology Support Plan under grant 2006BAJ03B06.

## References

- [1] Batterbee DC, Sims ND, Stanway R and Rennison, M (2007) Magnetorheological Landing Gear: 2. Validation Using Experimental Data, *Smart Materials and Structures*, 16(6). pp. 2441-2452.
- [2] Choi, SB, Lee, SK and Park, YP (2001) A hysteresis Model for Field-dependent Damping Force of a Magnetorheological Damper, *Journal of Sound and Vibration*, 245(2). pp. 375-383.
- [3] Choi, YT, Wereley, NM and Jeon, SY (2002) Semi-active Vibration Isolation Using Magnetorheological Isolators, *Proc. SPIE*. 4697. pp. 284-291.
- [4] Du, H, Lam, J and Zhang, N (2006) Modeling of a Magneto-rheological Damper by Evolving Radial Basis Function Networks, *Engineering Applications of Artificial Intelligence*, 19(8). pp. 869-881.

- [5] Dyke, SJ, Spencer, BF, Sain, MK and Carlson, JD (1996) Modeling and Control of Magnetorheological Dampers for Seismic Response Reduction, *Smart Materials and Structures*, 5. pp. 565-575.
- [6] Gao, GS, Yang, SP, Chen, EL and Ma BY (2004) Experimental Modeling and Its Application for Semi-active Control of High-speed Train Suspension System, *Chinese Journal of Mechanical Engineering*, 40(10). pp. 87-91. (in Chinese).
- [7] Gao, M and Wang CZ (2008) Inverse Modelling of MR Damper Based on ANFIS Technique and Its Application, *Journal of Vibration and Shock*, 27(3). pp. 140-142, 164, 185-186.
- [8] Guan, XC and Guo, PF (2009) Modeling of Magnetorheological Dampers Utilizing a General Nonlinear Model, *International Conference on Adaptive Structures and Technology*, Hongkong.
- [9] Jansen LM and Dyke SJ (2000) Semi-active Control Strategies for MR Dampers: Comparative Study, *Journal of Engineering Mechanics*, 126(8). pp. 195-803.
- [10] Jiang, N and Li, ZX (2008) Simulation Analysis of MR Damper Control Over Seismic Responses of Adjacent Structures, *Earthquake Engineering and Engineering Vibration*, 28(2). pp. 131-136. (in Chinese).
- [11] Kamath, GM and Wereley, NM (1997) Nonlinear Viscoelastic-Plastic Mechanism-based Model of An Magnetorheological Damper, *Journal of Guidance, Control and Dynamics*, 6. pp. 1125-1132.
- [12] Li, ZQ, Du, CB, Yu, GJ and Sun, LG (2009) Experiment Research on Damper Characteristics and Improved Damping Model of MRD, *Journal of Vibration and Shock*, 28(5). pp. 124-136. (in Chinese).
- [13] Ma, XQ, Wang, ER, Rakheja, S and Su, SY (2002) Modeling Hysteretic Characteristics of MR-fluid Damper and Model Validation, 41st IEEE Conf. on Dec. and Control, 2. pp. 1675-1680.
- [14] Peel, DJ, Stanway, R and Bullough, WA (1996) Dynamic Modeling of an ER Vibration Damper for Vehicle Suspension Applications, *Smart Materials and Structures*, 5(5). pp. 591-606.
- [15] Schurter, KC and Roschke, PN (2000) Fuzzy Modeling of a Magnetorheological Damper Using ANFIS. *Proceedings of IEEE International Conference on Fuzzy Systems*. pp. 122-127.
- [16] Sims, ND, Stanway, R and Peel, DJ, Bullough, WA and Johnson, AR (1999) Controllable Viscous Damping: an Experimental Study of an Electrorheological Long-stroke Damper Under Proportional Feedback Control, *Smart Materials and Structures*, 8(5). pp. 601-615.
- [17] Sims, ND, Peel, DJ, Stanway, R, Johnson, AR and Bullough WA (2000) The Electrorheological Long-stroke Damper: a New Modeling Technique with Experimental Validation, *Journal of Sound and Vibration*, 229(2). pp. 207-227.
- [18] Wang, JX and Meng, G (2001) Theoretical and Experimental Study on the Vibration Control by Magneto-rheological Fluid Dampers, *Journal of Vibration and Shock*, 20(2). pp. 39-46. (in Chinese).
- [19] Wang, DH and Liao, WH (2004) Modeling and Control of Magnetorheological Fluid Dampers Using Neural Networks, *Smart Materials and Structures*, 14(1). pp. 111-126.
- [20] Wang, ER, Ma, XQ, Rakheja, S and Su, SY (2005) Modeling hysteretic characteristics of an MR-fluid damper, *Proc. Inst. of Mech. Engrs. Part D: Journal of Automobile Engineering*, 217(7). pp. 537-550.
- [21] Wang, W, Xia, PQ and Liu CY (2006) Experimental Modeling of MR Dampers Based on Bouc-Wen Function, *Journal of Vibration Engineering*, 19(3). pp. 296-301. (in Chinese).
- [22] Wang, J, Zhou, CG, Zhu, CC, Xie S.L. and Zhang, XN (2007) Hybrid Modeling of Automotive Suspension System Using Magnetorheological Damper, *Engineering Mechanics*, 24(10). pp. 170-174. (in Chinese).
- [23] Wang, XJ and Gordaninejad, F (2007) Flow Analysis and Modeling of Field-controllable, Electro- and Magneto-rheological Fluid Dampers, *Journal of Applied Mechanics*, 74(26). pp. 13-22.
- [24] Wang XY, Song, C, Chen, ZQ, Sun, HX and Chen, PH (2009) Test of MR Damper and Model by Using Neural Network, *Journal of Vibration and Shock*, 28(4). pp. 42-46. (in Chinese).
- [25] Weng, J.S., Hu, H.Y. and Zhang M.K. (2000) Experimental Modeling of Magneto-Rheological Dampers, *Journal of Vibration Engineering*, 13(4). pp. 616-621. (in Chinese).
- [26] Wereley, N.M, Pang, L. and Kamath, G.M. (1998) Idealized Hysteresis Modeling of Electrorheological Dampers, *Journal of Intelligent Material Systems and Structures*, 9. pp. 642-649.
- [27] Wilson, CMD and Abdullah, M (2005) Structural Vibration Reduction Using Fuzzy Control of Magnetorheological Dampers. *Proceedings of the 2005 Structures Congress and the 2005 Forensic Engineering Symposium*, New York.



- [28] Xu, ZD, Li, AQ, Chen, WR and Ye, JH (2005) A Temperature Phenomenological Model With Mass Element of Magnetorheological Damper, *Engineering Mechanics*, 22. pp. 144-148. (in Chinese).
- [29] Yang G (2001) Large scale magnetorheological fluid damper for vibration mitigation: modeling, testing and control, Ph.D. Thesis, University of Notre Dame, Notre Dame, USA.
- [30] Zhou, Q, Nielsena SRK and Qu, WL (2005) Semi-active Control of Three-dimensional Vibrations of an Inclined Sag Cable with Magnetorheological Dampers, *Journal of Sound and Vibration*, 296. pp. 1-22.